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Hydraulic characterization of aquifers, reservoir rocks, and soils: A history of ideas

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Abstract. Estimation of the hydraulic properties of aquifers, petroleum reservoir rocks, and soil systems is a fundamental task in many branches of Earth sciences and engineering. The transient diffusion equation proposed by Fourier early in the 19th century for heat conduction in solids constitutes the basis for inverting hydraulic test data collected in the field to estimate the two basic parameters of interest, namely, hydraulic conductivity and hydraulic capacitance. Combining developments in fluid mechanics, heat conduction, and potential theory, the civil engineers of the 19th century, such as Darcy, Dupuit, and Forchheimer, solved many useful problems of steady state seepage of water. Interest soon shifted towards the understanding of the transient flow process. The turn of the century saw Buckingham establish the role of capillary potential in governing moisture movement in partially water-saturated soils. The 1920s saw remarkable developments in several branches of the Earth sciences; Terzaghi's analysis of deformation of water-saturated earth materials, the invention of the tensiometer by Willard Gardner, Meinzer's work on the compressibility of elastic aquifers, and the study of the mechanics of oil and gas reservoirs by Muskat and others. In the 1930s these led to a systematic analysis of pressure transients from aquifers and petroleum reservoirs through the work of Theis and Hurst. The response of a subsurface flow system to a hydraulic perturbation is governed by its geometric attributes as well as its material properties. In inverting field data to estimate hydraulic parameters, one makes the fundamental assumption that the flow geometry is known a priori. This approach has generally served us well in matters relating to resource development primarily concerned with forecasting fluid pressure declines. Over the past two decades, Earth scientists have become increasingly concerned with environmental contamination problems. The resolution of these problems requires that hydraulic characterization be carried out at a much finer spatial scale, for which adequate information on geometric detail is not forthcoming. Traditional methods of interpretation of field data have relied heavily on analytic solutions to specific, highly idealized initial-value problems. The availability of efficient numerical models and versatile spreadsheets of personal computers offer promising opportunities to relax many unavoidable assumptions of analytical solutions and interpret field data much more generally and with fewer assumptions. Currently, a lot of interest is being devoted to the characterization of permeability. However, all groundwater systems are transient on appropriate timescales. The dynamics of groundwater systems cannot be understood without paying attention to capacitance. Much valuable insights about the dynamic attributes of groundwater systems could be gained by long-term passive monitoring of responses of groundwater systems to barometric changes, Earth tides, and ocean tides.

> Many of the important details of history and interaction in science are often forgotten; we know about such-and-such an idea, that somebody had it at some point, but we don't know just how it happened, who said what to whom, and so on. Those things can be terribly important in the real development of science. So let me tell you something about the history of the maser and the laser.
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> *C. H. Townes* [1994, p. 53]

1. Introduction

In order to solve practical problems of interest in the fields of groundwater seepage, hydrogeology, agricultural engineering, petroleum engineering, environmental engineering, soil physics, and geophysics, it is necessary to have reliable estimates of hydraulic parameters such as permeability, hydraulic capacitance, and porosity. Since the early work of Darcy, Dupuit, Forchheimer, and others in Europe during the second half of the 19th century, a substantial body of literature has accumulated in diverse fields of Earth sciences and engineering pertaining to methods for estimating hydraulic characteristics by inverting data collected from experiments conducted on field installations. For a student of the Earth sciences it is of considerable interest not only to gain an understanding of how the ideas relating to hydraulic characterization have evolved historically but also to decipher the fundamental notions which unite all these methods.

It is reasonable to state that all the hydraulic characteriza-

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tion methods in use today have two themes in common: an empirical equation of motion, familiarly known as Darcy's law, which gives formal identity to the notion of permeability, and the equation of transient heat conduction, originally proposed by Fourier in 1807, which has established itself as the working model for diffusion-type processes in physical sciences. The equation of motion is imbedded in the diffusion equation.

Intrinsic to the transient diffusion equation (stemming from Fourier's equation of transient heat conduction) are the parameters hydraulic conductivity and hydraulic capacitance. In turn, hydraulic capacitance includes, among other properties, the porosity of the porous medium. The terms "storativity" and "specific storage" are often used in groundwater hydrology to denote hydraulic capacitance and specific hydraulic capacity of water-saturated geologic materials. For purposes of generality, we shall prefer, in this work, the term "hydraulic capacitance." This term includes storativity as a special case. Hydraulic capacitance represents the quantity of water released from storage because of a unit change in pressure due to a combination of three independent processes: pore volume change, change in water saturation, and expansion of water. The transient diffusion equation provides the foundation for hydraulic characterization. Ultimately, all the hydraulic characterization methods consist of fitting the field data to the transient diffusion equation and finding the best combination of parameters that agree with the field data. Thus hydraulic characterization methods are "inverse" methods concerned with the estimation of parameters compatible with the diffusion model.

In the inversion venture outlined above, Earth scientists and engineers have historically relied on the use of "analytic solutions" (also referred to as "closed-form solutions"). A variety of ingenious techniques (type-curve matching, early-time and late-time approximations) have been devised to back the parameters out from the field data. Taking advantage of developments in digital computers, researchers have, over the past two decades, been successfully experimenting with numerical models to estimate hydraulic parameters by way of "calibration" exercises. With the improvements in the reliability of solutions generated by numerical models and the increased availability of powerful "spreadsheets" on the personal computer, there are indications that numerical models will soon become preferred tools of inversion of field data to estimate hydraulic parameters. Numerical models are especially attractive because they can help minimize many assumptions that enter into the idealizations that are essential for obtaining closed-form solutions.

At present, as the personal computer drastically changes our approach to analyzing field data from hydraulic tests, it is worth our while to summarize our current knowledge of hydraulic characterization in a systematized manner and to assess where we stand. So motivated, the present work is an attempt to take an integrated view of concepts, ideas, and methods developed in agricultural engineering, soil physics, hydrogeology, petroleum engineering, civil engineering, geophysics, and related fields. This is a substantial task, considering the vast amount of literature that has accumulated on this topic over many decades, not to speak of the many differences that exist among these fields in articulating questions and describing ideas. Under the circumstances the goal of the present study, of generating an overall synthesized understanding of the field based primarily on literature from the United States, is a modest one. Even in this regard no claim is made that the literature compiled is comprehensive or complete. The hope is that the literature surveyed is adequate enough to capture the essential elements of the major ideas and concepts of relevance. *Hall* [1954] presented a well-reasoned review of literature on the topic of seepage towards wells. His survey is especially comprehensive in regard to the 19th century European literature. Hall's paper has been a valuable source of information in regard to the European literature discussed in the present work.

This work primarily focuses on field methods rather than laboratory methods. The hydraulic response of a subsurface flow system is governed by its geometric attributes, its forcing functions, and its material properties. In inverting field data to estimate hydraulic parameters, it is traditional to assume that the geometric attributes of the flow system and the forcing functions are known and the hydraulic parameters are the unknowns to be estimated. This work is restricted to those methods in which geometric details (symmetry, layering) are assumed known a priori. Over the past two decades an increasing body of literature has accumulated on the application of stochastic methods and probability concepts to the hydraulic characterization of pervasive heterogeneities (especially hydraulic conductivity variations within a homogeneous medium) in subsurface flow systems. These methods are outside the scope of the present work.

The history of science is such that ideas are born and methods are fabricated in response to pure curiosity or practical needs. Sometimes new ideas are born by accident or by inspiration. Integration of ideas to identify underlying unity among diversity comes later. Individual disciplines in the Earth sciences have generally been focused on problems of special interest to their needs. For example, until recently, soil physicists have devoted much of their attention to the process of infiltration into soils and the movement of water in the root zone. Thus, although the various Earth disciplines have a common thread of unity in terms of physical processes governing hydraulic characterization, these disciplines have traditionally maintained distinct identities, with limited flow of ideas among themselves. A consequence is that when one attempts an integration of ideas reaching beyond disciplinary boundaries, as in the present work, the portrayal of the individual disciplines cannot be very even. This is an unavoidable human limitation.

In the following the evolution of ideas is presented in a chronological fashion. As a prelude to the details that are to be presented, Figure 1 shows the important developments since 1807, and Figure 2 shows the important publications pertaining to the developments shown in Figure 1.

2. Darcy in Context

Darcy's law has come to play such a central role in the study of flow of fluids in the Earth's subsurface that Darcy's work gives the appearance of the start of a new era. However, Darcy came upon the scene at a time when mathematical physics was blossoming in France in the wake of the Newtonian revolution. Thus it is instructive to place Darcy's work in a historical context.

Our conceptual model for understanding the occurrence and movement of fluids in geological materials is based on treating fluid flow as a process mathematically analogous to heat conduction in solids. As a consequence, the working mathematical model for the transient flow of fluids in geologic materials is the partial differential equation of heat conduction, originally proposed by Fourier (Theorie de la propagation de la Chaleur

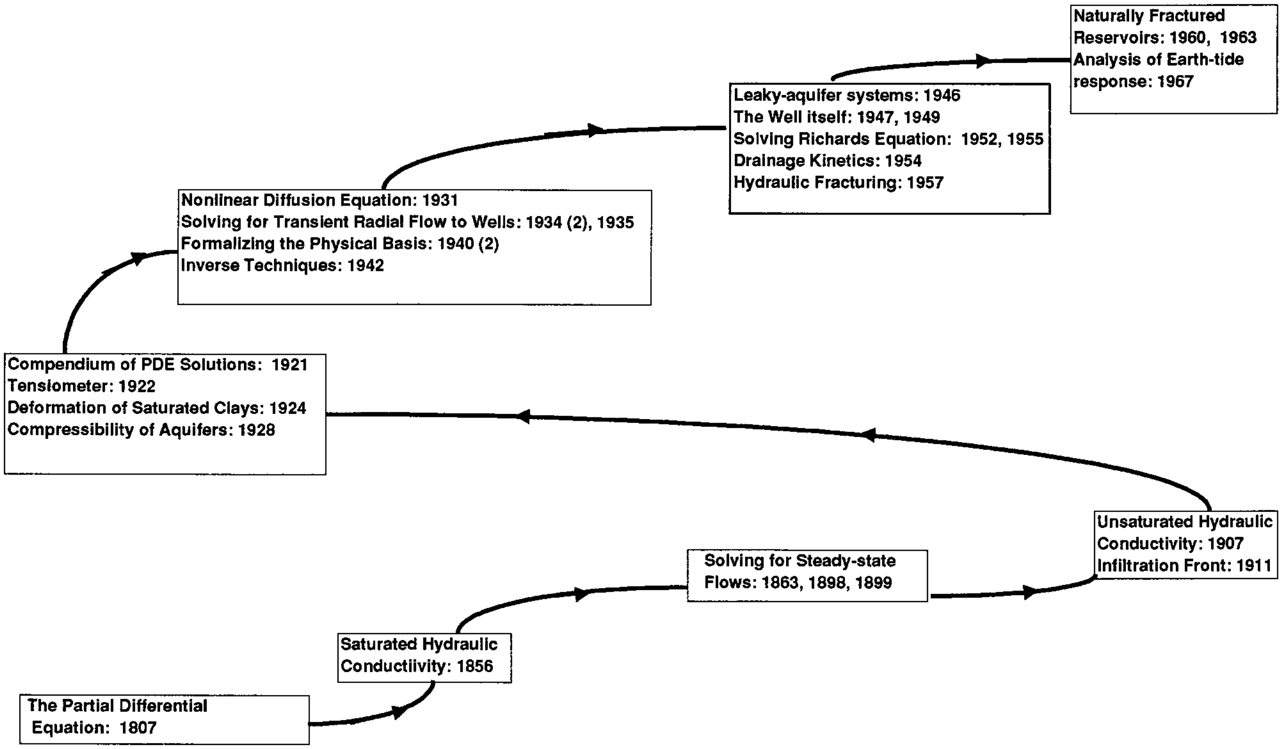

Figure 1. Chronology of important developments.

dans les solides, manuscript submitted to the Institute de France, 1807). As it happened [*Grattan-Guinness*, 1972], Fourier's 1807 work was never formally published. After much additional work to answer criticisms of the reviewers (Laplace, Lagrange, Monge, and Lacroix), Fourier's classic *Theorie Analytique de la Chaleur* was published in 1822. Intrinsic to Fourier's model are the parameters conductivity and capacitance. The development of these two concepts in the study of heat are

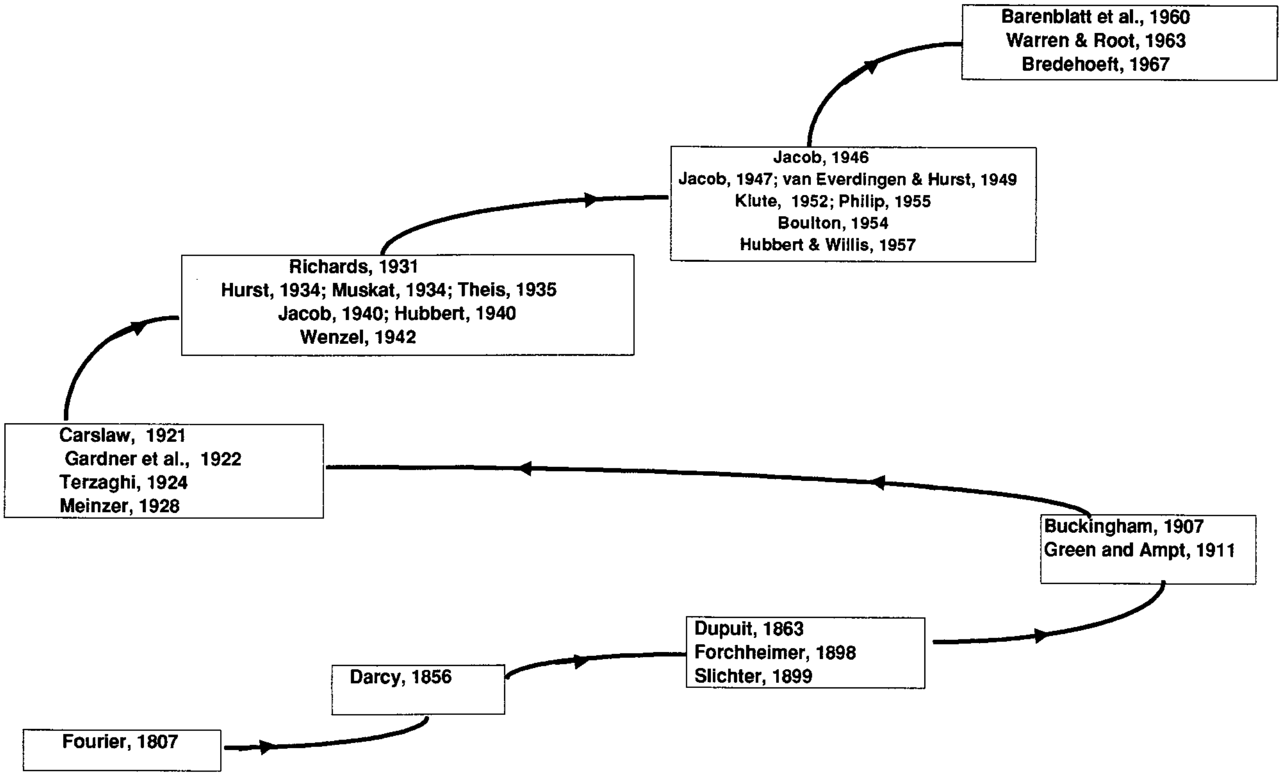

Figure 2. Chronology of important papers.

of fundamental scientific importance, and it is pertinent to start with a brief look at their history.

The construction of reliable thermometers for the precise measurement of heat was critical to the development the science of heat during the second half of the 18th century. Although the mercury thermometer was first constructed in France by Boulliau in 1659, accurate thermometers with well-defined scales became available only by the middle of the 18th century through Fahrenheit in 1724 and Celsius in 1742 [*Cajori*, 1898]. With the availability of these instruments, Joseph Black, a pioneer of modern quantitative chemistry at Edinburgh, discovered latent heat and specific heat, devised an ice calorimeter, and measured these quantities around 1760. However, he did not publish his work. The first published measurement of the specific heat of solids is attributed to a collaborative memoir by Lavoisier and Laplace ("Memoire sur la Chaleur") presented to the Royal French Academy in 1783. Thus the origin and definition of the concept of heat capacity is well documented. Fourier, in his 1807 manuscript on the propagation of heat, introduces the conductivity parameter (he calls it "specific internal conducibility") in precise mathematical terms. Since heat flux is difficult to measure without a calorimeter, it is reasonable to suspect that the measurement of thermal conductivity became established after the successful use of the calorimeter by Lavoisier and Laplace in 1780.

Other noteworthy developments were taking place in the sciences during the 18th century and the first half of the 19th century. In his most important work, *Hydrodynamica*, published in 1738, Daniel Bernoulli of Switzerland identified the three components that make up the mechanical energy of a moving fluid; potential energy due to gravity, elastic energy due to fluid pressure, and kinetic energy. Working with direct current, *Ohm* [1827] experimentally determined the inverse relationship between electric current and voltage drop along an electric conductor, the constant of proportionality being the electrical resistance of the conducting body. This electrical resistance was found to be a function of the material of the conductor as well as its shape and size. *Poiseuille* [1842], a medical doctor interested in the mechanisms of blood flow through human and animal veins, studied the mechanisms governing the flow of liquids through fine capillary tubes. These meticulous experiments were carried out on horizontal capillary tubes, and fluxes as low as 0.1 cm^3 over several hours were measured. Through these experiments Poiseuille established that the volumetric flux of liquid was directly proportional to the pressure drop over the tube and the area of cross section and inversely proportional to the length of the tube, the constant of proportionality being represented by K. According to *Herschel* [1940], the commonly used expression for Poiseuille's law involving the fourth power of capillary radius was not proposed by Poiseuille himself. Rather, the function was derived by several workers, including James Clerk Maxwell, by integrating Newton's equation for viscosity as applied to a cylindrical tube. In Germany *Hagen* [1839] had experimentally obtained similar results.

It is easy to see from the foregoing that by the time Darcy ventured into conducting his experimental studies on sand filters for water supply to Dijon [*Darcy*, 1856], a well-defined framework was already in place for the design of the experiment and the interpretation of the results, thanks to the contributions of Fourier, Ohm, and Poiseuille. One could reasonably assume that Darcy was aware of these developments and made use of them in his work. Darcy's work was unique in that it dealt with a natural material rather than an engineered material, such as a capillary tube, and that he accounted for gravity. Here again one could reasonably assume that Bernoulli's work in the field of hydraulics had influenced Darcy in including gravity in the definition of fluid potential.

It is pertinent here to take note of the difference between the mathematical forms of Darcy's law and Ohm's law. In Ohm's law one considers the resistance of the conducting body as a whole. Resistance, as it appears in Ohm's law, is an integral, evaluated over the body as a whole. Darcy's law, on the other hand, has a form exactly the same as Fourier's law for conductive heat flux and involves spatial derivative of potential. Whereas Darcy's law casts experimental observations in a form that is compatible with the differential equation, Ohm's law, it appears, is inherently well suited for integral equations and flow nets.

3. Second Half of the 19th Century

Immediately following Darcy's insightful contribution, analogy to heat conduction was actively pursued by engineers in Austria, France, and Germany to solve practical problems of groundwater seepage that were of interest to civil engineers during the second half of the 19th century. Although Fourier's general equation addressed the transient heat conduction process, these civil engineers restricted themselves to steady state flow systems. Whereas the transient process involves two parameters (conductance and capacitance), the steady state problem involves only the conductivity parameter.

The following discussion of the development of ideas in Europe during the 19th century is based on work by *Hall* [1954], who reviewed the European literature in considerable detail.

Jules-Juvenal Dupuit, a contemporary of Darcy, was a theoretically oriented civil engineer who dealt with problems of open channel flow as well as seepage through soils. The chapter on seepage in his book on open-channel flow [*Dupuit*, 1863] later proved to be a standard reference on the subject. It is interesting that Dupuit, starting from the hydraulic principles of open-channel flow, derived an expression for movement of water through soils that proved to be equivalent to Darcy's empirical law. By integrating the equation of motion over a radial domain, Dupuit derived solutions for steady flow in a confined aquifer (artesian well) and in an unconfined aquifer (gravity well). He idealized the well to be at the center of a circular island so as to satisfy the mathematical needs of a credible boundary condition. The assumption of horizontal flow he made in the case of a gravity well in an unconfined aquifer, while yet accounting for the variation in the saturated thickness of the aquifer, is used even now and is referred to as the Dupuit assumption. In addition to analyzing well hydraulics, Dupuit also studied the role of wells in regional groundwater systems such as artesian basins. Another important figure of this period was Joseph Boussinesq, who, in investigating the role of friction in laminar flow of fluids (liquids and gases), derived the expression for the flow of water in an idealized parallel-plate fracture, now referred to as the cubic law [*Boussinesq*, 1868].

In Germany, Adolf Thiem and, later, his son, Gunther Thiem, carried out pioneering work on groundwater seepage, especially in the study of the flow of water to wells. They are also credited with the collection of extensive observational information on the subject. Although he later became aware of the contributions of Dupuit and Darcy, Adolph Thiem independently derived the expressions for the steady radial flow of water in confined and unconfined aquifers. In the field of groundwater hydrology, Gunther Thiem [*Thiem*, 1906] is widely known for the equation describing the steady radial flow of water in a confined aquifer, although that solution was derived earlier by Dupuit. Gunther Thiem distinguished himself by systematizing and documenting the application of field methods rather than by creating new methods himself.

At this juncture it is appropriate to briefly digress and discuss terminologies. The word "groundwater" ("grundwasser" in German) appears in the literature by the early 1880s in the work of Adolph Thiem. The engineers of the late 19th century distinguished between "gravity wells" and "artesian wells." The former term referred to wells in a phreatic aquifer whose upper boundary is a free surface or the water table over which the pressure is atmospheric. The latter term referred to what is currently recognized as a confined aquifer. Although the possibility of a seepage face above the water level in a gravity well was recognized, the term "seepage face" had not yet been coined. Aquifers were commonly referred to as "groundwater streams" [*Hall*, 1954].

Perhaps the most well known researcher of this era was Phillipp Forchheimer of Austria, whose distinguished career spanned nearly a half century and influenced the work of many who followed him. He was among the earliest to recognize the concepts of isopotential lines and streamlines with regard to groundwater seepage and extended these concepts systematically to generate flow nets as a means of quantitatively analyzing steady flow fields, including flow of water to wells under varying geometric conditions. Forchheimer formally wrote down the Laplace equation [*Forchheimer*, 1898] to describe the steady flow of groundwater and went on to use mathematical techniques such as conformal mapping to solve problems. It appears [*Hall*, 1954] that he was influenced by the work on conformal mapping of *Holzmüller* [1882] for the solution of heat conduction problems. In addition to formally explaining the results of earlier workers such as Dupuit and Thiem, Forchheimer presented new results for single wells as well as groups of wells and sloping aquifers.

In the United States *Slichter* [1899] pioneered the study of groundwater systems by mathematically analyzing the steady flow of water through geologic media. In particular, he investigated mutual interference between artesian wells and the perturbation of the regional steady state groundwater flow field by a producing water well. Slichter was unaware of Forchheimer's work and formulated the Laplace equation independently. He obtained solutions using the conformal mapping method. It appears that Slichter too, like Forchheimer, was influenced by the work of Holzmüller. Another important contribution of Slichter was that he investigated the physical significance of hydraulic conductivity, which was merely treated as an empirical coefficient by Darcy. By studying the geometric properties of various spherical packs, Slichter identified the geometric component and the viscous drag components of hydraulic conductivity.

4. The Turn of the 20th Century

In retrospect it is easy to see that our understanding of the flow of fluids through porous media during the 19th century was largely due to the contributions of researchers from France. The turn of the century saw the center of activity in relation to the study of water movement in earth materials gradually shift from Europe to North America.

Edgar Buckingham, a leading physicist of the time, was a member of the Physical Laboratory of the Bureau of Soils, U.S. Department of Agriculture (USDA), during the first decade of this century. His insightful studies on the movement of moisture in soils resulted in two fundamental contributions, published as bulletins of the USDA. In the first, devoted to aeration in soils, he formulated [*Buckingham*, 1904] the parabolic equation to describe the movement of gases in unsaturated soils and evaluated the time lag and damping associated with the migration of air driven into and out of the soil because of changes in barometric pressure. Recognizing the fact that earlier workers such as Dupuit, Forchheimer, and Slichter addressed the steady state problems, Buckingham appears to be the first worker to address the transient migration of fluids in the Earth's subsurface. Buckingham's second bulletin, published in 1907, proved to be far reaching in impact [*Buckingham*, 1907]. Studying the movement of moisture in unsaturated soils theoretically and experimentally, he defined capillary potential and proposed an equation similar in form to Darcy's law to describe the movement of moisture in unsaturated soils. A special feature this formulation was that hydraulic conductivity was treated as a function of capillary potential rather than simply being a constant, as in Darcy's law. The full scope of Buckingham's contribution would become possible later with the invention of the tensiometer. It is remarkable that Buckingham, who was probably not aware of Darcy's work [*Sposito*, 1987], gave a theoretical basis for Darcy's empirical law and extended the law to the unsaturated zone. Because Darcy's law is now used widely for both the saturated zone and the unsaturated zone, some would argue that Darcy's law should be properly called the "Darcy-Buckingham law." Buckingham's work is central to the fields of soil physics and multiphase flow analysis still in use today. *Buckingham* [1914] is also widely known for his seminal contribution on dimensional analysis. Most of the methods used to invert field data to obtain hydraulic parameters on the basis of analytic solutions routinely use dimensionless groups to minimize the number of variables that need to be handled. The rationale for defining these dimensionless groups stems from the "pi theorem" proposed by Buckingham in 1914.

The early 20th century saw the simultaneous recognition of the importance of time in many Earth science disciplines: soil science, soil mechanics, groundwater hydrology, and petroleum engineering [*Narasimhan*, 1986, 1988]. Researchers in these fields recognized that almost all subsurface fluid flow systems are dynamic in nature. In the field of soil physics *Green and Ampt* [1911] proposed a simple approximation to quantify the vertical infiltration of water into an unsaturated soil. The Green and Ampt idealization assumes that as water infiltrates into a soil, a sharp, piston-like zone of saturation advances with time. This approximation is still used in interpreting field data from infiltrometer tests to estimate in situ hydraulic conductivity of soils.

5. The Remarkable Era: 1920–1940

In the evolution of ideas pertaining to the flow of fluids in geological media, the period 1920–1940 must rank as truly remarkable. Conceptual as well as mathematical contributions of fundamental importance relating to the dynamic behavior of fluid flow systems were made simultaneously in the fields of soil physics, petroleum engineering, groundwater hydrology, and civil engineering. Through a combination of laboratory studies, field observations, and mathematical analysis, the foundations were laid during this period for the systematic study of the dynamics of subsurface flow system.

Willard Gardner was among the earliest [*Gardner and Widtsoe*, 1921] to attempt to quantify nonsteady moisture movement in unsaturated soils in terms of a transient diffusion equation analogous to Fourier's transient heat conduction equation. It is now known that his failure to achieve satisfactory agreement between experiment and theory was due to the fact that he did not account for the dependence of hydraulic conductivity on capillary potential, suggested a decade earlier by Buckingham. In other words, he tried to fit experimental data to a linear partial differential equation, when in fact a nonlinear parabolic equation should have been used.

The early 1920s saw the publication of the classic book *Introduction to the Mathematical Theory of Conduction of Heat in Solids* [*Carslaw*, 1921]. This book and its revision [*Carslaw and Jaeger*, 1947] constituted a remarkably well-organized compendium of a variety of closed-form solutions to problems in steady state and transient heat conduction. The availability of these solutions and the methods used to derive these solutions have proved to be of great benefit to Earth scientists and engineers over the past 75 years in solving a host of fluid flow problems of the Earth's subsurface.

The 1920s saw the publication of two major contributions in the Earth sciences focusing attention on physical processes. *Terzaghi* [1924] experimentally studied the deformation of water-saturated clays and established the relationships among external stresses, pore-fluid pressure, and deformation. In the process, he introduced the important notion of effective stress. Some would consider Terzaghi's paper to have founded the discipline of soil mechanics. Terzaghi proceeded to write down and solve the equation for transient movement of water in a one-dimensional clay column by analogy with the heat conduction equation. In his paper, Terzaghi was meticulous in establishing the one-to-one correspondence between the attributes of the heat-conduction system and the porous-medium flow system. Probably he was the first to point out that the compressibility of a clay is conceptually analogous to specific heat of a solid.

A second major contribution of the 1920s was the paper by *Meinzer* [1928], whom many would consider to be the founder of the discipline of groundwater hydrology in the United States. Meinzer's distinctly descriptive paper was a careful synthesis of observations by many geologists of the U.S. Geological Survey (USGS) of the early 20th century who had studied the decline in water pressures in artesian aquifers such as the Dakota sandstone in North Dakota because of overdraft. These observations, based on water budget calculations, led to the inference that the declines in water pressures were correlated with a decrease in porosity and an increase in water volume, which together accounted for the mass of water mined from the aquifer. Considering the fact that the strains so caused in the porous medium and in the water are extremely small (less than one part in a million), it was remarkably perceptive of Meinzer and his coworkers to have drawn their inferences on the basis of rough estimates of water balance. *Meinzer* [1937], in fact, had made rough estimates that as a result of the decrease in porosity, the land in the North Dakota artesian basin should have subsided by 4–5 inches (10.2–12.7 cm), and this was viewed with skepticism by contemporary geologists and engineers. In essence, Meinzer had described the physics of hydraulic capacitance in saturated geologic materials, which would be described as "storage coefficient" by Theis a decade later.

Contemporaneously with Terzaghi's new leadership in the field of soil mechanics, important developments were also taking place in the analysis of seepage through soils. *Forchheimer* [1930] published his book *Hydraulik*, and *Dachler* [1936] published his book on groundwater flow, which contained a host of steady seepage problems, including flow to wells. For the first time a successful attempt was made by *Weber* [1928] to analyze the nonsteady flow of water to a gravity well (that is, nonsteady flow to a fully penetrating well in an unconfined aquifer). The approach taken by Weber to analyze this problem is worth some discussion because it differs significantly from the more rigorous mathematical approach of later workers such as Muskat, Hurst, and Theis, who solved a parabolic partial differential equation.

Weber considered a well in which the water level is maintained at a constant (constant drawdown test). As pumping progresses, the radius of the cone of depression (also referred to as the radius of influence) increases with time. As a first step in the analysis of this problem, Weber derived an approximate expression for the radius of influence, assuming that water is released from storage by physical drainage of the volume of the aquifer through which the water table moves and that the volume of water so drained per unit volume is the "effective porosity" (the modern notion of specific yield). Mass balance requires that the volume of water so drained is equal to the cumulative production at the well. Once the effective radius is estimated, drawdown as a function of distance from the well is estimated from the steady state solution of radial flow to a gravity well. About a decade later, similar results were obtained by *Steinbrenner* [1937] in Austria.

By the late 1920s the tensiometer had become well developed, thanks to the efforts of Willard Gardner and his coworkers [*Gardner et al.*, 1922]. This work, an abstract, is reportedly the first published reference to the tensiometer (W. Gardner, personal communication, 1991), an instrument that has played a vital role in the evolution of modern soil physics. Because of the tensiometer, routine measurements of moisture content and its relation to capillary pressure had become possible [*Richards*, 1928]. Combining *Buckingham*'s [1907] work on the equation of water motion in unsaturated soils with the newly available soil moisture retention curves, *Richards* [1931] formally wrote down, for the first time, the nonlinear partial differential equation describing transient flow of water in unsaturated soils. The slope of the moisture content versus capillary pressure curve came to be the hydraulic capacitance and was referred to as "moisture capacity." Because of the difficulties of obtaining closed-form solutions to nonlinear differential equations, Richards equation remained unsolved for nearly two decades. It would be the early 1950s before *Childs and Collis-George* [1950] showed that the severity of nonlinearity of the parabolic equation could be lessened by using volumetric moisture content as the dependent variable, rather than capillary potential. Following this suggestion, *Klute* [1952], *Philip* [1955], and others began obtaining solutions for Richards' equation under highly simplified conditions using numerical methods.

In the field of petroleum reservoir engineering, the 1930s was an eventful decade. The need for applying rigorous methods of mathematical physics to understanding the dynamics of oil and gas reservoirs had been recognized. The decade started with careful theoretical and experimental study of steady state flow systems as a prelude to the study of transient systems which followed immediately thereafter. *Muskat and Botset* [1931] experimentally studied the steady flow of gases in geologic materials and verified that the mass flux of gas was proportional to the drop in the square of the pressure along the flow path. This dependence on the drop in the square of pressure for gases had been theoretically derived by *Boussinesq* [1868]. They then went on to formulate the nonlinear parabolic equation for transient gas flow in a reservoir and solved the special case of steady radial flow in a circular reservoir with a well at the center and a constant pressure outer boundary. *Wyckoff et al*. [1932], with the help of physical models, experimentally studied the radial flow of water in a sand body with a free surface (an unconfined aquifer) and verified the assumptions of Dupuit. They also extensively discussed the importance of the seepage face, the capillary fringe, and water movement in the unsaturated zone above the water table.

The notion of estimating reservoir permeability from transient field tests was initiated by *Moore et al*. [1933]. They articulated a need for estimating, from field tests, important properties of reservoir rocks so that the drainage of oil reservoirs could be studied. This little-known work is very significant for many reasons. Although detailed mathematical derivations were not presented, the authors formally laid down the parabolic equation involving a slightly compressible fluid, presented solutions for a well of finite radius producing at constant pressure from a finite cylindrical reservoir, calculated drawdown and buildup, and demonstrated how the solution can be made use of to estimate reservoir permeability. Furthermore, the authors presented their results in terms of the two important dimensionless groups, dimensionless time and dimensionless drawdown. These dimensionless groups have since become part of the petroleum engineering and groundwater hydraulics literature. At about the same time, *Muskat* [1934] presented a detailed analysis of transient flow of compressible fluids in oil and gas reservoirs. He derived solutions for wells of finite radius as well as of vanishingly small radius in a circular reservoir with prescribed potential boundary or with prescribed flow rate at the well. He then went on to establish the veracity of his model with pressure decline data from an oil field in east Texas. *Hurst* [1934] formulated the parabolic equation in radial coordinates for slightly compressible fluids (liquids) and obtained solutions for production at constant pressure and at constant discharge from a well of finite radius, pumping a cylindrical reservoir of finite radius. Both Hurst and Muskat considered hydraulic capacitance arising purely from fluid expansion and neglected changes in porosity. Shortly afterward, *Muskat* [1937] published his definitive work on the flow of homogeneous fluids through porous media in which he elucidated the fundamental problems of modern petroleum reservoir engineering and the mathematical methods for solving them.

In the field of groundwater hydrology, *Theis* [1935] set up and obtained a solution to the parabolic equation similar to that of *Hurst* [1934] and *Muskat* [1934] but considered a laterally infinite aquifer with a well of vanishingly small radius (line-source well) producing at a constant rate. He verified the credibility of his model by applying it to field data from an unconfined aquifer. Theis used the term "storage coefficient" to denote the hydraulic capacitance parameter in the parabolic equation, a term which still enjoys common usage. Although he was quite cognizant of the analogy between heat capacity and hydraulic capacitance [*Freeze*, 1985], Theis did not explicitly discuss the physical meaning to storage coefficient in his paper. It appears that Theis took a fairly limited view of storage coefficient, restricted to the particular boundary value problem he was interested in, namely, a laterally infinite aquifer of finite thickness, in which water flows horizontally. Thus *Theis* [1940] explains storage coefficient as the volume of water released from a vertical prism of the aquifer of unit cross-sectional area in response to a unit change in hydraulic head. Moreover, *Theis* [1940] identified the role of compressibility in regard to storage coefficient in an artesian aquifer but did not recognize expansion of water. This restricted view of storage coefficient came to enjoy popular usage among groundwater hydrologists in the USGS in subsequent decades.

Theis' work has proved to be a milestone not only in groundwater hydrology but in the Earth sciences in general. In addition to constituting the basic and simplest technique used widely for interpreting data from transient aquifer tests, the Theis' model is also frequently used as the standard against which the transient behavior of more complex aquifers is studied for comparison. One of the factors contributing to the popularity of Theis' work appears to be the fact that hydrologists of the USGS actively developed workable techniques for practically applying the Theis' solution to interpret field data from aquifer tests. They widely communicated their results through USGS publications, which are readily available to field geologists. In addition, these contributions provided a large-scale regional perspective of hydraulic characterization in terms of Earth processes in general. In contrast, contributions in the fields of civil engineering, petroleum engineering, and soil physics took a limited local view of the characterization venture. A landmark publication in this regard was USGS Water Supply Paper 887, by *Wenzel* [1942], which elaborately described the various methods for interpreting pumping test data.

It is worth noting here that *Moore et al*. [1933], *Muskat* [1934], and *Hurst* [1934] were all concerned with laterally limited reservoirs, whereas groundwater hydrologists such as *Theis* [1935] were concerned, in general, with laterally infinite systems. Also, petroleum engineers concentrated on developing techniques for analyzing data from the production well, whereas groundwater hydrologists devoted attention to pumped-well analysis as well as analysis of interference test data (that is, data from passive, observation wells which respond to the removal of water at the pumped well). More than one reason can be attributed to these differences in the styles of design and analysis of hydraulic tests between petroleum engineers and groundwater hydrologists. According to W. E. Brigham (personal communication, 1996) petroleum engineers generally had to work with active well fields, in which many wells were producing fluids at the same time. Under such conditions, planes of no-flow boundaries developed between producing wells, leading to the dynamic isolation of each well. Groundwater hydrologists, on the other hand, did not often deal with well fields. Moreover, the occurrence of oil is very commonly associated with dissolved natural gas and, as the pressure drops during production, gas tends to come out of solution. A consequence is that the apparent compressibility of such oil may be orders of magnitude higher than gas-free oil, leading to a great increase in the effective hydraulic capacitance. In turn, increased hydraulic capacitance contributes to a relatively small radius of influence around a production well, and hence there is reduced need for interference analysis. Another possible explanation for the differences in styles between petroleum engineering and groundwater hydrology is that petroleum reservoirs often constitute closed systems, while groundwater systems are in general open in nature.

Upon reflection, it is evident that the notion of capacitance is essential for describing the transient flow process. In the work of *Gardner and Widtsoe* [1921], hydraulic capacitance was restricted purely to the rate of change of saturation with capillary pressure, referred to as moisture capacity in the soil physics literature. In Terzaghi's work, hydraulic capacitance was restricted solely to the compressibility of a relatively soft porous material for which one could reasonably neglect the compressibility of water. Meinzer's work combined porous medium compression and fluid expansion in giving form to hydraulic capacitance. *Hurst* [1934] and *Musket* [1934] restricted hydraulic capacitance solely to expansion of the fluid [*Narasimhan*, 1986, 1988]. In general, in a saturated-unsaturated deformable porous medium hydraulic capacitance includes all the three components, namely, pore-volume change, change in water saturation, and expansion of water [*Narasimhan and Witherspoon*, 1977].

At present, it is almost invariably assumed by hydrogeologists that the Theis method is applicable to confined aquifers in which water release from storage is due to the elastic properties of the porous medium and of water. However, it must be noted that in his classic paper, *Theis* [1935] applied his method to an unconfined aquifer and stated that "the equation applies rigidly only to water bodies . . . and [is] applicable only to unconfined water bodies—in which the water in the volume of sediments through which the water table has fallen is discharged instantaneously with the fall of the water table" (p. 521). However, it had been recognized by previous workers that the drainage of water in an unconfined aquifer is an extremely complex physical process. Therefore, as noted by *Hall* [1954], the instantaneous drainage assumption of Theis is a shortcoming of the Theis method as applied to unconfined aquifers.

Arthur Casagrande is a respected name in the field of soil mechanics. Although he did not publish many papers on the theory of flow to wells, *Hall* [1954] notes that commencing from 1934, Casagrande introduced his students at Harvard University to novel ideas in regard to seepage theory, including the flow of water to wells. As part of his lectures, Casagrande had demonstrated that for large values of time, the drawdown predicted by Weber's method and that predicted by Theis' method are essentially the same.

In the field of civil engineering a little known but major discovery, made in the early 1930s, was to influence the attention of Earth scientists and engineers for the next half a century. *Rappleye* [1933], of the U.S. Coast and Geodedic Survey, carefully documented substantial "areal subsidence" of land in the Santa Clara Valley of California based on rerunning of first-order leveling surveys during 1931–1932. He reported that between 1920 and 1933 a benchmark in San Jose had subsided by 4.1 feet (1.25 m) and that as much as 0.5 feet (0.15 m) of that subsidence had occurred during 1932–1933. Although heavy groundwater pumpage was suspected to be the cause of the subsidence [*Tibbetts*, 1933], it was left to *Meinzer* [1937] to advance a rational physical mechanism correlating groundwater pumpage and observed land subsidence. Meinzer not only recognized the applicability of his North Dakota observations [*Meinzer*, 1928] to the San Jose subsidence, but he also conjectured that the substantial magnitude of subsidence observed was probably due to a preponderance of soft, fine-grained sediments in the Santa Clara basin. As we shall see later, Meinzer's conjecture was confirmed subsequently by meticulous field observations by Poland and coworkers in the Santa Clara Valley and the San Joaquin Valley of California.

6. The War Years: Formalizing the Foundations

The decade of the 1940s was quite eventful in the study of transient groundwater systems. *Hubbert* [1940] published "The Theory of Ground-Water Motion," a paper that still remains definitive. In this paper Hubbert elaborated the physical meaning of a fluid potential, formally defined permeability on the basis of balance between impelling forces and resistive forces, derived a tangent law for the refraction of flow lines, and went on to establish the foundations for the study of regional groundwater systems and petroleum reservoirs.

We saw earlier that *Theis* [1940] took a restricted view of storage coefficient limited to horizontal flow in an elastic aquifer. However, *Jacob* [1940] took a much more fundamental view of storage coefficient in the sense of hydraulic capacitance and derived an expression combining the deformability of the porous medium (its bulk modulus) and the compressibility of water. He thus gave formal identity to the processes heuristically recognized by *Meinzer* [1928, 1937]. Jacob also went on in this classic paper to derive an expression for the change in water pressures in aquifers subjected to external stress changes, such as those caused by passing trains, and to barometric pressure changes and defined the parameter "tidal efficiency." Jacob's theoretical work paved the way for interpreting hydraulic parameters of aquifers by analyzing these responses. Jacob went on to make two other major contributions during the 1940s.

7. Post–World War: Building the Superstructure

By the early 1940s the physical and mathematical foundations for transient movement of fluids in subsurface geological systems involving porous medium deformation, desaturation, and fluid expansion had been clearly formalized. The succeeding half a century, extending to the present, has seen the application of these basic ideas and tools to field conditions involving a wide variety of flow geometry, initial conditions, and forcing functions.

Jacob [1946] extended Theis' method to heterogeneous media when he published a paper on radial flow to a leaky aquifer, which opened up a fertile area of research relating to multiple-aquifer systems in groundwater hydrology and petroleum engineering. It is not quite clear whether Jacob was influenced by the land subsidence research of the 1930s. It is now well established that the study of leaky aquifer systems and the study of land subsidence in sedimentary basins go hand in hand. Also, motivated by a desire to account for the role of the well itself, *Jacob* [1947] devoted attention to hydraulic efficiency of the well, as water dynamically flows from the aquifer into the well. He defined the notion of effective well radius and the well-loss function. In accounting for well losses, Jacob accounted for nonlaminar flow conditions arising because of high flow velocities in the vicinity of well screen. Such flows are some times referred to as "non-Darcy" flows. In the field of petroleum engineering, *van Everdingen and Hurst* [1949] used the Laplace transformation to quantify the effects of well bore storage on pressure transients around a pumping well and also accounted for skin effects arising from formation damage in the immediate vicinity of the well.

These developments in the fields of hydrogeology and petroleum engineering occurred primarily because the researchers mentioned above were interested in aquifers and reservoirs with fairly large areal extent lying at depths of a few hundred meters or more. In such formations the region of pressure perturbation around the well often extended to several hundred meters or more. However, in the fields of soil physics and civil engineering, transient flow problems of interest were of a smaller spatial scale. Soil scientists and agronomists were primarily interested in the plant root zone of the soil above the water table, which seldom exceeds a few meters from the land surface. Civil engineers and geotechnical engineers on the other hand were interested in seepage and ground settlement problems extending from a few meters to perhaps a few tens of meters. The nature of problems tackled by these researchers was such that they needed to estimate hydraulic parameters rather quickly and inexpensively. Soil physicists dealing with soils in the vadose zone not only were confronted with significant spatial variability on the scale of their observation but also had to contend with a very difficult-to-solve highly nonlinear diffusion process. The term "vadose zone" denotes the region between the water table and the land surface within which water and air coexist in the pore spaces. It is also referred to as the "zone of aeration" or the "unsaturated zone." Out of practical necessity, judicious compromise between mathematical rigor and practical need gave rise to greatly simplified models, resulting in field techniques based on infiltrometers, constant-head permeameters, auger-hole tests, and variable-head permeameters.

The auger-hole methods and piezometer methods were pioneered by Kirkham and coworkers [*Kirkham*, 1946; *Luthin and Kirkham*, 1949; *van Bavel and Kirkham*, 1948]. These methods improved the estimation of the hydraulic conductivity of the saturated soil below the water table and are still being used. Essentially, these are field adaptations of the variable-head permeameter. Although the experiment itself involves a nonsteady flow process, the interpretation logic neglects the role of hydraulic capacitance. The time-dependent falling water level is treated as a function, among other factors, of the hydraulic conductivity of the soil and a shape factor dependent on the flow geometry. Because the flow geometry involved combinations of radial, hemispherical, and vertical components of flow, a great deal of effort was spent by Kirkham and others to calculate shape factors for a variety of field conditions. Thus, calculating the shape factors using available mathematical techniques constituted an important part of developing these techniques.

As in the case of soil science, the variable-head permeameter was found to be adequately inexpensive and rapid to satisfy the hydraulic characterization needs in the field of civil engineering. Special efforts were made to systematize and standardize these methods. A widely used work in this regard was that of *Hvorslev* [1951], published under the auspices of the U.S. Army Corps of Engineers. In providing a set of shape factors for a number of field situations, Hvorslev drew upon earlier work of Dachler and others.

Richards, who formulated the nonlinear partial differential equation for flow in unsaturated soils was known for his many innovative experiments. *Richards et al*. [1956] demonstrated a method by which the hydraulic conductivity function could be estimated in the field by measuring the depth profile of gauge pressure head as well as moisture content as a function of time during redistribution of soil moisture immediately following an infiltration event. In this experiment soil moisture distribution was measured rather laboriously by a gravimetric method. A powerful and useful development of the early 1950s in the field of soil physics was the use of neutron scattering to quantitatively estimate soil moisture [e.g., *Gardner and Kirkham*, 1952]. Soon this developed into a workable field neutron probe, which continues to play an important role in the measurement of soil moisture profile in the field for research and engineering purposes. During the 1960s the field method of Richards and his coworkers was improved by other researchers by taking advantage of the neutron probe.

Another important work in soil science during the 1950s was that of *Gardner* [1957]. Because of the nonlinearity of Richards' equation, it was evident that inversion of field data on unsaturated hydraulic conductivity would not be possible unless some simple relationship between hydraulic conductivity and gauge pressure head could be assumed. On the basis of available data, *Gardner* [1957] found that over a limited range of gauge pressure, one could reasonably assume an exponential relationship between hydraulic conductivity and gauge pressure head. Gardner then went on to point out that an exponential relation helps to obtain closed-form solution for the one-dimensional problem. This work of Gardner and the work of *Philip* [1955] continue to influence present day research relating to hydraulic characterization of unsaturated soils in the field.

A significant contribution of the 1950s was the work of N. S. Boulton, a civil engineer from England. As was noted earlier, *Theis* [1935] illustrated the credibility of the transient groundwater flow equation by applying it to data gathered from an unconfined aquifer. Nevertheless, as Theis himself recognized, his method hinged on the assumption that water drains instantly from the zone through which the water table declines. However, it was recognized by many that the drainage of water from the unsaturated material above the water table, governed by the theory of capillary potential, was a time-dependent, noninstantaneous process. This time-dependent drainage is mathematically analogous to chemical disequilibrium processes such as precipitation or adsorption. Therefore it is reasonable to refer to the noninstantaneous drainage of water from the vadose zone as "kinetically controlled drainage." Some researchers felt the need to mathematically account for this noninstantaneous process. *Boulton* [1954] initiated investigation of the transient flow of water to a well in an unconfined aquifer. Instead of venturing to rigorously solve the highly complex flow process above the water table as embodied in Richards' equation, Boulton simplified the effect of the unsaturated zone by introducing the approximation of delayed yield in conjunction with the notion of specific yield. As an approximation, he assumed that drainage from the unsaturated zone was an exponential function of time. The resulting governing equation was solved for potentials within the saturated domain, while yet approximately accounting for contribution from the unsaturated zone by means of a time-dependent source term. With minor modifications and extensions, Boulton's model still continues to be used by groundwater hydrologists as the basis for estimating parameters of an unconfined aquifer.

Another important contribution of the 1950s was the work by *Skempton* [1954]. A soil mechanician, Skempton investigated the relations between external stress changes (including shear) and the changes in pore fluid pressure in water saturated soils. Skempton proposed pore pressure coefficients A and B, which are related in principle to the concept of tidal efficiency proposed earlier by Jacob but which account for multidimensional deformation and pertain to the effects of mean principal stress (coefficient B) and the effects of shear stress (coefficient A). The basis for estimating the hydraulic parameters of an aquifer from passive response of wells to barometric tides, Earth tides, and ocean tides is contained in the contributions of *Jacob* [1940] and *Skempton* [1954].

Soon after the publication of *Theis*' [1935] work, groundwater hydrologists developed several approaches to interpret drawdown data as well as data on water level recovery after cessation of pumping. Although groundwater hydrologists were routinely using Theis' recovery method for over a decade, it was not until the 1950s that petroleum engineers developed methods to systematically analyze pressure buildup (or pressure recovery) data. Research in this direction was pioneered by *Horner* [1951] and *Miller et al*. [1950]. Incidentally, it appears that modern pressure transient analysis in petroleum engineering commenced after the second world war, during the 1950s.

By now, the field of groundwater hydrology had become well enough established, and a definitive textbook on groundwater hydrology was published by *Todd* [1959]. This book devoted considerable attention to groundwater hydraulics and presented a comprehensive literature on the topic. Roger de Wiest brought to the western world some of the developments in the erstwhile Soviet Union by translating *Theory of Groundwater Movement* [*Polubarinova-Kochina*, 1952].

The 1960s witnessed many and varied developments of significance to hydraulic characterization. A group of USGS groundwater hydrologists, led by Hilton Cooper, elegantly extended the Theis approach to solve many well-defined initial value problems that have since enabled hydraulic characterization under test conditions that are more general than those of *Theis* [1935]. Among these contributions one should take special notice of the interpretation of data from slug tests [*Cooper et al*., 1967], analysis of pressure transient data from an anisotropic aquifer [*Papadopulos*, 1965], transient flow of water to a well of large diameter [*Papadopulos and Cooper*, 1967], and response of a well to seismic waves [*Cooper et al*., 1965]. The work on seismic response showed how a well could, under certain conditions, amplify a seismic signal. Following this work, *Bredehoeft* [1967] analyzed the response of aquifers to Earth tides, giving consideration to multidimensional strains experienced by an aquifer and proposing a method for estimating the storage coefficient (hydraulic capacitance) of an aquifer. This work continues to be widely used to interpret passive response of aquifers to Earth tides.

The 1960s briefly witnessed an interest in the use of electrical analog models to analyze the transient behavior of groundwater systems, including the flow of water to wells. Although the use of electrical network models was know in other branches of science, *Skibitzke* [1963] recognized the possibility of using resistor network models to simulate the transient behavior of wells in aquifers, with electrical resistance being analogous to hydraulic resistance and electrical capacitance being analogous to hydraulic capacitance. This idea was soon pursued by researchers in USGS. *Papadopulos* [1965] used such a model to verify his analytical solution for the flow of water in an anisotropic aquifer, while *Bredehoeft et al*. [1966] used an harmonic oscillator circuit in conjunction with resistors and capacitors to simulate inertial effects of water column movements in a finite diameter well subject to periodic loading. The interest in electrical analog models soon waned with the rapid development in digital computer technology during the late 1960s.

The study of leaky aquifers, pioneered by Jacob a decade earlier, was continued with vigor by Hantush and Jacob through the 1960s. Because they were primarily concerned with groundwater as a resource, Hantush and Jacob focused attention on analysis of drawdown data from the aquifer itself and did not venture into obtaining solutions for changes in potential within the aquitards which constituted the source of leakage. *Hantush* [1964] provided a comprehensive summary of developments related to leaky aquifers as well as other aquifer configurations in the paper "Hydraulics of Wells."

The leaky aquifer problem attracted the attention of petroleum engineers from two different perspectives. On the one hand, engineers were aware of leakage of oil into reservoir rocks from leaky caprocks. On the other hand, they were also interested in the role of leaky caprocks in the context of artificial storage of natural gas in deep aquifers. In the latter case it was critical that the "integrity" of the caprock and its ability to keep the gas trapped in the aquifer be known. This necessitated a knowledge of the pressure changes in the aquitard itself rather than just the aquifer. Accordingly, *Neuman and Witherspoon* [1969] extended the leaky aquifer model of Jacob and Hantush to hydraulically characterize the aquifer as well as the aquitard.

Immediately following the discovery of land subsidence in the Santa Clara Valley [*Rappleye*, 1933; *Meinzer*, 1937] Joseph Poland of the USGS started a systematic study of land subsidence in different parts of California. Over the next four decades Poland and coworkers collected a wealth of data confirming *Meinzer*'s [1937] conjecture about the importance of fine-grained sediments in contributing to large subsidence magnitudes. Their work also established the applicability of Terzaghi's one-dimensional consolidation theory to large-scale geologic systems. *Poland and Davis* [1969] documented these observations in a classic paper. Note that from a process point of view, land subsidence is a manifestation of the hydraulic capacitance parameter.

During the 1960s the movement of oil in fractured reservoirs attracted the attention of petroleum engineers for two different reasons: the depletion of naturally fractured reservoirs and the pressure response of reservoirs stimulated by hydraulic fracturing. The analysis of flow in naturally fractured reservoirs received significant impetus from the work in the former Soviet Union of *Barenblatt et al*. [1960], who proposed a model for the dynamic, macroscopic interactions between a pervasive high-diffusivity continuum (fracture network) embedded in which are islands of low-diffusivity continua (porous rock matrix). The work of Barenblatt et al. was extended formally to the study of petroleum reservoirs with idealized fracture networks by *Warren and Root* [1963]. The conceptual basis provided by Warren and Root is still widely used in the fields of petroleum engineering and hydrogeology. The frequently referred to phrases "double-porosity systems," "dual-porosity systems," and "multiple-interacting continua" derive their existence from the work of Barenblatt et al. and of Warren and Root.

By the 1960s stimulation of low-permeability reservoirs by hydraulic fracturing had become commonplace in petroleum production engineering. Through an elegant analysis of the mechanics of hydraulic fracturing in an elastic rock, *Hubbert* *and Willis* [1957] showed that massive hydraulic fractures tend to manifest themselves as planar vertical fractures or horizontal fractures depending on ambient tectonic stress conditions. It became immediately clear that such high-permeability planar fractures will profoundly perturb the radial flow field around the production well. *Prats* [1961] was among the earliest workers to investigate the effects of discrete vertical fractures on the steady flow of oil into a well. Soon the analysis was extended to transient flow conditions by *Scott* [1963] and *Russell and Truitt* [1964].

In the field of geophysics the technique of hydraulic fracturing, originally developed for petroleum reservoir stimulation, was perceived as a means of estimating in situ rock stresses through “min-frac” experiments. Drawing upon the theoretical foundations of Hubbert and Willis and of *Kiehle* [1964], *Haimson and Fairhurst* [1969] and others pioneered work in this direction.

A significant research direction of the 1960s was the development of numerical models. The era of the digital computer had dawned, and computer development was advancing with incredible rapidity. The digital computer provided the possibility of solving transient fluid flow problems in complex geological systems which are far beyond the reach of closed-form solutions. The finite element method [*Clough*, 1960], which was initially designed for solving structural engineering problems, was soon adapted to solve steady state and transient problems of groundwater flow [*Javandel and Witherspoon*, 1968]. In the field of petroleum engineering *Fayers and Sheldon* [1962] illustrated the use of a digital computer to solve fluid flow problems in three dimensions using the classical finite difference approximations. In the field of civil engineering *Tyson and Weber* [1964] presented an integral form of the finite difference method which could efficiently handle groundwater systems with complex geometry. One of the important upshots of the development of the numerical model was the effort to hydraulically characterize the field system on the basis of observed water levels in numerous wells. Hydraulic characterization is achieved by a process of trial and error adjustment of hydraulic parameters in a numerical model to best match the field data. This approach to hydraulic characterization is popularly referred to as the “inverse method.” Inverse methods, stemming from this approach, continue to engage the attention of researchers today.

8. The 1970s: A Shift in Emphasis

The 1970s witnessed a shift in research emphasis among Earth scientists from issues based on resource development to issues related to environmental degradation. Research on topics introduced in the previous decades was continued, but new issues pertaining to chemical contamination began to be introduced. The delayed drainage concept of Boulton (in relation to unconfined aquifers) was questioned by *Neuman* [1972], who invoked vertical anisotropy instead of delayed drainage to account for the pressure transient behavior of unconfined aquifers. In keeping with emerging interest in environmental issues, strong research interests continued in improving methods for characterization of shallow groundwater systems and the vadose zone, in particular, slug tests, permeameters, and infiltrometers. In the field of petroleum engineering, considerable interest continued on the characterization of naturally fractured reservoirs. In order to better understand hydraulic properties of the vadose zone, *Weeks* [1978] devised a field method for evaluating pneumatic conductivity and diffusivity of the vadose zone based on transmission of barometric pressure changes from the land surface to the water table. Note that this possibility was inherent in the early work of *Buckingham* [1904].

During the 1980s groundwater contamination arising from leaky gasoline tanks of gas stations and contamination arising from the uncontrolled disposal of industrial hydrocarbons such as lubricants, transformer oils, and cleaning fluids came into unexpectedly sharp focus. Also, as a potentially serious health hazard, attention was given to the entry of radon gas into human dwellings in regions of the United States underlain by granitic rocks. In late 1987 the Congress of the United States decided that unsaturated zone disposal of high-level radioactive wastes at Yucca Mountain in Nevada would be the preferred geologic disposal alternative and that detailed site characterization studies should be carried out there before licensing. As a result, there has been a great impetus among researchers to develop techniques for characterizing the hydraulic properties as well as the pneumatic properties of the vadose zone.

Until the 1980s hydraulic characterization of soils by agricultural engineers was by and large limited to measuring the saturated hydraulic conductivity below the water table using auger hole tests, piezometer tests, and permeameter tests pioneered by Kirkham and others. The 1980s saw notable effort among soil physicists to apply already established theoretical results to devise instruments and methods to estimate hydraulic characteristics of unsaturated soils in the field. The theoretical basis for these efforts was already implicit in the studies of *Childs and Collis-George* [1950], *Philip* [1955, 1956], *Gardner* [1957], and others. The Guelph permeameter [*Reynolds and Elrick*, 1985] was a constant-head permeameter designed for small unlined boreholes a few meters deep and designed to estimate saturated hydraulic conductivity as well as the matric flux potential. The latter is an integral of the unsaturated hydraulic conductivity, between the limits of ambient pressure head in the vicinity of the borehole and zero pressure head. The 1980s also saw the development of disc tension permeameters [*Clothier and White*, 1981; *White and Perroux*, 1987; *Perroux and White*, 1988]. Designed for measuring the vertical hydraulic conductivity of the soil at the land surface, this instrument was especially designed to apply a constant moisture tension boundary condition at the land surface to enable infiltration at a water potential less than atmospheric. The disc permeameters were used in infiltration experiments carried out on the basis of the theoretical analysis of infiltration from circular ponds and under prescribed moisture suctions imposed at the disc at the land surface and measuring the infiltration rates. In essence, these are constant-head permeameters, except that a constant moisture suction is imposed. Interpretation of data is incumbent on several idealizations; for example, it is often assumed that hydraulic conductivity is exponentially related to moisture suction. In addition to saturated hydraulic conductivity, the disc permeameters enabled the estimation of sorptivity. In systems involving one-dimensional infiltration, sorptivity is a measure of the quantity of moisture absorbed by a one-dimensional column as a function of the square root of time.

The 1980s also saw active research designed to understand the role of water in influencing natural earthquakes. To aid in the interpretation of these field experiments, researchers extended *Bredehoeft*’s [1967] work to interpret the response of aquifers to barometric tides, Earth tides, ambient changes in tectonic stresses, and earthquakes.

The past 25 years have witnessed significant changes in the motivation for hydraulic characterization as well as the approaches used for the purpose. Interest in resource development has been accompanied by an increasing interest in mitigating and preventing the contamination of natural resources. There has been a growing desire to identify geological formations of very low hydraulic conductivity in which toxic wastes can be safely disposed. As a consequence, topics such as leaky aquifers and unconfined aquifers have gradually receded from researchers' focus of attention. Interest has been steadily growing in characterizing flow processes in the vadose zone, which mediates between the wastes deposited at the land surface and the water table at depth. Methods are being developed to quantify the movement of air, gases, and vapor through the vadose in addition to moisture movement. The dynamic coupling between gases in the vadose zone and atmospheric pressure changes is proving to be of considerable practical interest. To gain an understanding of local flow fields which control dispersion of contaminants, sensitive flowmeters are used by some researchers to get a profile of flow velocities with depth in boreholes and wells.

Major multimillion dollar hydraulic characterization ventures have been supported by the U.S. Department of Energy (DOE), the U.S. Nuclear Regulatory Commission (NRC), the U.S. Environmental Protection Agency (EPA), and others to hydraulically characterize heterogeneities in simple aquifers, in fractured rock systems, and in unsaturated media. Current emphasis is on characterizing the details of heterogeneity at different scales because such detailed information is necessary for quantifying the migration of contaminant plumes. As attempts are made to physically describe the heterogeneities in greater and greater detail, it is being realized that the traditional methods based on the differential equation are inadequate. For example, attempts to characterize fractured rock systems through interference tests and tracer tests have shown that on the scale of observation carried out, these systems can hardly be treated as homogeneous media.

9. Concluding Remarks

Presently, hydrogeologic systems are studied by researchers with interests varying widely, from the shallow soils in which plant roots thrive to the oceanic crust beneath the deep oceans in which hydrothermal fluids circulate. The motivation for these studies vary from engineering design to pure curiosity. To study these systems, field instruments of great sophistication (e.g., pressure transducers, sensitive flowmeters) of unprecedented precision have been fabricated. Automatic data loggers enable us to acquire data at frequencies of less than a second. Powerful desktop computers enable us to collect, store, retrieve, and rapidly manipulate enormous amounts of data. Yet it is reasonable to state that our ability to interpret data lags behind our ability to collect them. It is not uncommon to collect data at great expense and interpret the same with extremely simplified models, doing little justice to the system complexities, for no reason other than that no other methods of analysis are available. Or vast amounts of data may be gathered and simply stored because necessary resources and time are not available for interpretation.

The traditional basis of our data interpretation, the partial differential equation, is a statement of physical processes of cause and effect which are related through the model parameters. As we continue to use this model for hydraulically characterizing subsurface flow systems, we are confronted with, on the one hand, questions pertaining to the intrinsic worth of the cause-effect model itself and, on the other hand, questions relating to the information that is fed to the model for purposes of analysis. In the present work we have restricted ourselves to data inversion from hydraulic tests on systems with explicitly defined heterogeneities (e.g., leaky aquifers, double porosity systems). Each material component has been assumed to be homogeneous on the scale of observation. We do not concern ourselves with pervasive heterogeneities within a homogeneous medium, information on whose geometric attributes are not known in sufficient detail.

Because the concepts of homogeneity, geometry, and continuity are essential for properly posing a problem with the differential equation, one has to set up one separate differential equation for each of the components in a heterogeneous system and couple them together at the interfaces between the components. Thus, for each component of the heterogeneous system, we must have an appropriate macroscopic scale in which the component can be considered homogeneous so that macroscopically defined parameters have physical meaning. In the case of hydraulic tests (be it a disc tension permeameter on the soil surface or an interference test between wells separated by kilometers) the scale is dictated by the scale of the experiment and the geometric details of the system are taken to be known at that scale based on geological, geophysical, and other information. Therefore one of the intrinsic limitation of our methodology is that field data cannot be inverted to estimate parameters on a smaller scale for which geometric details are lacking. Even when geometric details of the flow system are known on the scale of the field test, nonuniqueness of the estimated set of parameters is often to be expected because the number of parameters to be estimated may exceed the number of observation sites at which observations have been made. Finally, devices such as boreholes and wells, which enable the measurement to be made, themselves disturb the natural system and fundamentally influence the measurement that is being made. In order that the intrinsic hydraulic properties of the formation are evaluated, the masking influence of the influence of the device on the measurement must be carefully removed.

Looking purely from the viewpoint of science, it may appear as though what stands between us and satisfactory characterization is adequate data. Often we are limited by resource availability for data gathering, be it a research venture or engineering venture. Even in those situations where sufficient resources are available, one must consider whether the wells or boreholes themselves may compromise the integrity of the site. Constrained by these practical concerns, we need to recognize that the Earth's subsurface is difficult to access. Consequently, our methods of hydraulic characterization are only capable of yielding estimates, and we function on the reasonable premise that the estimates become more reasonable with better information and fewer assumptions in the interpretation process.

Insofar as interpretation of data from field hydraulic tests is concerned, we can hope to improve over our traditional approach of fitting the data to analytic solutions. As we have noted, the flow system has to be considerably idealized before particular solutions can be obtained for the partial differential equation. Numerical models have certainly arrived at a point at which the configuration of a field test can be given full consideration in a numerical model, thereby minimizing the assumptions that otherwise go into an analytic solution. With the availability sophisticated graphics softwares, numerical models have the potential of helping us design complex field experiments, the data from which can be inverted in a credible fashion. Also, if modern tomographic methods that are being developed in the field of geophysics are successful, we may in fact have access to hitherto unavailable geometric detail which can be very valuable in data inversion.

Currently, the issue of heterogeneity pertaining to hydraulic conductivity is particularly in focus among many researchers. This interest has arisen because of the nature of certain field problems engaging the attention of these researchers (e.g., efficient application of irrigated water to the roots of cultivated crops, the prediction of the behavior of contaminant plumes, and the secondary recovery of oil by water flooding). It is worth noting here that hydraulic conductivity is only one of the two parameters of fundamental importance in dynamic subsurface flow systems. Hydraulic capacitance is an equally important parameter. Indeed, we cannot conceptualize the transient groundwater flow system without introducing hydraulic capacitance. The timescale of response of a groundwater body is dictated by its diffusivity, which is hydraulic conductivity divided by specific hydraulic capacity. By excessively focusing attention on hydraulic conductivity, one may overlook transient phenomena that are of great intellectual and practical interest.

Passive monitoring of the response groundwater systems to changes in barometric pressures, earth tides, ocean tides and changes in tectonic stresses can yield information of great value about the hydraulic capacitance as well as the hydraulic conductivity of the system. With the availability of electronic data loggers, passive monitoring can be used to great advantage in gathering information which is distinct from and complementary to information gathered from conventional hydraulic tests. Much remains to be done to systematize the interpretation of these data. It is even likely that long-term passive monitoring may yield information on the gradual change in system properties over long periods of time.

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