Raymond D. Mindlin—A Bio/Bibliographical Sketch

Raymond David Mindlin was born on September 17, 1906, in New York City, the second of three sons of Henry, a prosperous businessman, and Beatrice (nee Levy). There are clear indications that the family, which subsequently came also to include a half-sister, was supportive and closely knit, and that Ray, as he came to be known to his friends and colleagues, enjoyed a tranquil childhood.

For his secondary education the young man was sent to the highly regarded Ethical Culture School, of which he had fond recollections in later years. Upon graduating in 1924 he enrolled at Columbia, beginning an association with the University that was to last for more than half a century. Four years later (during which time he distinguished himself as a sprinter on the varsity track team) he received the first of his four earned degrees, a B.A., followed by a B.S. in 1931, and in 1932 by a C.E. and the Illig medal for "proficiency in scholarship."

The economic depression then gripping the land was a major reason why he remained for graduate work, funded by the modest stipend that accompanied the research assistantship to which he was appointed. Love's *Treatise on the Mathematical Theory on Elasticity*, the fourth and last edition of which had appeared not long before, came to his hand, and from then on the direction of his professional work was firmly set.

Applied mechanics was not strongly represented on Columbia's faculty; in fact, there was little substantive research in it in the United States, and essentially no graduate instruction, until the advent on the scene of Stephen Timoshenko, exiled by the Bolshevik revolution in Russia. Some time after his appointment as professor at the University of Michigan in 1927, he organized a series of summer courses, in which each year one or another of the foremost names in this field, including L. Prandtl, R. V. Southwell, and H. M. Westergaard, participated. Mindlin attended in 1933, '34, and '35, and there is no doubt that the experience at Ann Arbor served to confirm him in his choice of his life's work.

His initial publication [1][1] dates from the middle of this period, a solo, full-length paper describing a new type of polariscope for photoelastic analysis. April of 1935 saw the appearance of a lengthy discussion, written with Westergaard, of a paper on torsion of structural beams. During the year, Mindlin spent all available time in his basement office struggling to master Love's grand opus, a massive, difficult, demanding book, from time to time having to contend with complaints, from a faculty lacking in theoretical training and outlook, that the young man was wasting time. It was James Kip Finch, his Department chairman (and in later years a noted historian of engineering as well as Dean of the Engineering School), who served both as his protector and source of encouragement by issuing an edict to his colleagues, "Leave him alone, maybe something will come of it." Mindlin, in his turn, never forgot Finch's kindness and support, and it was at his persistent urging that the University, 35 years later, created the Finch chair (with Mindlin its first occupant).

For his doctoral research Mindlin set himself a fundamental problem in theoretical elasticity: determining the stresses in an elastic half-space subjected to a sub-surface point load. Working without any guidance at Columbia, he succeeded in finding the solution by employing the method of nuclei of strain. The results, nowadays referred to as "Mindlin's problem," represent a

[1] Numbers in brackets refer to corresponding entries in the list of publications (pages xxi-xxviii).

generalization of the two classical 19th century solutions respectively associated with the names of Kelvin and Boussinesq, and have become the basis for analytical formulations widely employed in geotechnical engineering. At Timoshenko's urging, in order to establish the author's priority of discovery, a summary of the results was dispatched to the *Comptes Rendus* of the *Académie des Sciences* in Paris, noted for almost instantaneous publication, where it appeared [3] in September of 1935. The full paper [4] was published in *Physics* (now the *Journal of Applied Physics)* in 1936, the year Mindlin received the Ph.D. degree. It was also the year in which he published his one and only paper to appear in a periodical devoted to pure mathematics, in essence a completeness proof of the Papkovich representation in classical elasticity [S]. It is interesting to note that nearly twenty years later he returned to the problem bearing his name and offered a solution by means of a systematic procedure grounded in potential theory 1421.

Academic advancement, it appears, proceeded at a rather leisurely tempo in those days. Mindlin remained an assistant for another two years, at which point he was elevated to instructor in civil engineering, and only in 1940 did he receive promotion to assistant professor. His research in the six years following receipt of the doctorate was a mix of analytical and experimental endeavors, with emphasis on the photoelastic method of stress analysis. It included a work on three-dimensional photoelasticity [17], published jointly with D. C. Drucker, his first doctoral student, in which the method of oblique incidence was first suggested. An important part of current laboratory technique is based on this and a companion and two subsequent papers 1181, C261, C271.

In 1942 Mindlin was co-opted by the Applied Physics Laboratory in Silver Spring (a Maryland suburb of the nation's capital), an institution engaged in naval ordnance work. There he played a significant role in the development of the proximity fuse, one of the major achievements in the scientific war effort. For his part in its success he was presented, shortly after the end of the war, with the Presidential Medal for Merit, the highest decoration awarded to civilians.

The swiftness of his climb up the academic ladder now made up for the earlier glacial pace: he came back to Columbia in 1945 as an associate professor, and two years later attained the rank of professor.

The year of his return to academia also marked the publication, in the *Bell System Technical Journal,* of a small monograph on the dynamics of package cushioning [21], which quickly became a vade mecum for designers of containers for protection of electronic equipment from mechanical shock and vibration. It was the first of several papers stimulated by his eight-year association as consultant to the Bell Telephone Laboratories.

A most important paper arising from his symbiosis with the Bell Laboratories stemmed from a perennial effort toward rational design of the carbon microphone. Published in 1949, it gave new direction to research on frictional phenomena at surfaces in contact [28]. The work, its basic consequences first tested and verified experimentally [38] and then generalized [41], led to an understanding of the process of wear due to local sliding and gave a mighty impetus to world-wide researches, analytical and experimental, in the industrially vital areas of sliding and rolling contacts, ball bearings, railway wheels, and the mechanics of granular media [56].

Shortly thereafter Mindlin began his trail-blazing studies, which he was to continue intermittently for more than two decades, on waves and vibrations in isotropic elastic plates [34] and, concommitantly, on high-frequency vibrations of crystal plates [35], soon extending his studies to account for the piezoelectric effect [39]. The method he had employed of reducing the exact, three-dimensional equations to describe the dynamics of plates, by expanding in series of the thickness coordinate (or in terms of selected functions thereof) and truncating the series, he also applied, suitably altered, to dynamical problems of bars [37], [60], [62], stimulating, as in the case of plates, an ever growing literature of the subject.

The work on isotropic bars and plates led to results in use world-wide in development and design of electromechanical filters and solid delay lines. But it was his studies on crystal plates that caused the greatest stir, for they were pioneering papers in a vexingly difficult mathematical field. In them, he elucidated a very complicated phenomenon of great technical importance and thereby pointed the way to major improvements in the design and performance of quartz crystals for filter circuits. The U.S. Army Signal Corps, long-time sponsor of the research on crystal plate vibrations, persuaded him to prepare a monograph on the subject by granting him a sole-supplier contract for the purpose. In 1955 the task was completed to the acclaim of the sponsor-it was in the form of a ca. 170-page-long report entitled "An Introduction to the Mathematical Theory of Vibrations of Elastic Plates." Yet even as its author was composing the text, better ways of doing the old were occurring to him and new ideas for what should, and could, be done were springing to his mind. Indeed, despite much subsequent work in other areas of mechanics, he kept returning to this subject to the very end of his life. The monograph seemed to haunt him, for he intermittently worked on a revision-more accurately, a complete reworking-of it. A few months before his death he spoke of plans to add yet another major part to what he had already written before he would consider the job done; ever the realist, he added, "But don't hold your breath!"[2]

It is strikingly clear in retrospect that the period of the late '40s and early '50s was a particularly fruitful one for him. In addition to the basic papers on elastic contact and on high-frequency vibrations of plates and bars, a pioneering article on the interpretation of optical birefringence in viscoelastic materials [27] and a seminal paper on the interaction of an elastic shell with a surrounding fluid [40], the second of these written in collaboration with his colleague H. H. Bleich, date to that time. Of evident importance to the Navy, the latter work triggered a great deal of subsequent research on the response of submarine hulls to shock loading. Also sandwiched into that period is the lengthy article on analogies [31], with another of his colleagues, M. G. Salvadori, as co-author, which appeared as a chapter in the *Handbook of Experimental Stress Analysis.*

Beginning in 1962, in response to what purported to be, but were seen by Mindlin as erroneously based, expanded theories of anisotropic materials, he was led to construct, first, a theory that took account of couple-stresses [74], characteristically applying it immediately thereafter to a two-dimensional case that seemed simple enough to be susceptible of experimental validation [75]. Two years later, in a paper on micro-structure [78], he treated a continuum endowed at each point with an internal displacement field, and showed that the lowest order of this formulation results in a theory that includes the optical lattice modes at the long-wavelength limit. In a paper published the following year [83], he formulated a linear theory of equilibrium of an elastic solid in which the stored energy depends on the strain and its first and second gradients. This proved to be the first continuum description capable of ac- counting for surface tension, without resorting to the artificial construct of a vanishingly thin surface membrane. The papers contributed to the start of lively activity in this field of generalized elastic continua, and stimulated applications to areas as diverse as the mechanics of laminated and of fiber- reinforced materials, and of framed structures.

[2] Shortly after making this observation, aware of the gravity of what proved to be his last illness, he entrusted the manuscript for completion (and, it is fervently hoped, timely publication) to P. C. Y. Lee, one of his last doctoral students.

In 1967 he was appointed James Kip Finch Professor of Applied Science, the first holder of a chair named in honor of the patron and protector of his days as a graduated student. He was to hold this post until his retirement in 1975.

A series of papers initiated with [88] introduced the contribution of the polarization gradient (in addition to the strain and the polarization itself) to the stored energy of a solid. Mindlin was able to show that this augmented formulation of the elastic dielectric continuum accommodates the surface energy of deformation and polarization, predicts the experimentally observed anomalous capacitance of thin dielectric films [95], and accounts for acoustical and, when the magnetic field is included, optical activity [99].

More or less concurrently came one other group of highly innovative researches, in which his aim was to bridge the gap between continuum and lattice theories. Thus, he was able to show [91] that a model of a continuum, formed by two or more interpenetrating media, in which the stored energy depends not only on the strains of the individual constituents but also on their relative displacements, and simulating a (non-Bravais) lattice with a basis, contains optical as well as acoustical branches in the vibrational spectrum. In [95], a paper mentioned above, he demonstrated that his augmented theory of the elastic dielectric continuum yields the correct long-wave limit of a monatomic lattice theory with a shell model, and in [loll, by endowing the model of [9l] with individual polarizations, that the resulting continuum can yield the long-wave limit of a diatomic lattice with shell model.

The steady stream of his research was interrupted early in 1969 by illness, and he was obliged to submit to cardiac surgery. (This was before the coronary bypass operation became part of clinical procedure.) As he made amply clear at the time, he resented the lengthy convalescence because it was inhibiting his work. But by the end of that year he was back at his desk, and a glance at his list of publications after that date shows that he was able to resume his enviable scientific productivity.

His last hurrah, just a year before his death, came in the form of two papers, one dealing with free vibrations of a rectangular parallelepiped [128] and the other, with vibrations of a rectangular plate with all four edges free [129], classic problems against which he had butted his head, in vain, decades earlier. At long last he managed to construct solutions, apparently by inspired guess- work, reminiscent of the *tour de force* of his doctoral dissertation 50 years earlier. He was manifestly elated, particularly by what turned out to be his final publication, characterizing the accomplishment in the following words: “It’s a pity there aren’t many around, anymore, who would appreciate what an extraordinary feat that solution is.” His letter, accompanying a photocopy of the work in holograph and dated January 9, 1986, continues, in evident allusion to the 19th century origin of the problem, “I’m reminded of Tony Biot [Maurice Anthony Biot] who was frequently a generation ahead of his time. I, on the other hand, seem to be some generations behind.”

The preceding narrative, though sketchy, has, I hope, given some indication of the breadth and depth of Mindlin’s scholarship. Those who worked with him, and those others who knew him and his work, quickly came to appreciate his profound insights into the physics of any given situation, his surefootedness in discerning what is central and what is peripheral, and his multifaceted analytical and experimental skills. Nevertheless, a disservice would be done the subject of this account (as well as its reader) if nothing were said about him as colleague, mentor, and human being.

His professional colleagues treated him with deference at times bordering on awe. He was secure in the knowledge of his own worth, but wore the mantle of his eminence with genuine modesty. He was generous in giving or sharing credit, and unfailingly courteous to peers seeking his opinion or advice. Basically a shy, reserved person, he was, invariably and unexceptionally, the consummate gentleman.

His many disciples revered him and, one suspects, in his quiet way he enjoyed being the father figure. To celebrate his 60th birthday, his former doctoral students tendered him a dinner in Minneapolis, where most had journeyed ostensibly to attend a professional meeting. In 1974, in anticipation of his imminent retirement, he was presented with a published book, entitled *R. D. Mindlin and Applied Mechanics.* In eight substantial chapters, by fifteen of his erstwhile students, this novel kind of Festschrift contains summaries of those areas on which Mindlin's own endeavors have exercised a profound and lasting effect.

But it was by no means a one-way relationship. Whenever a former student of his was being feted, as such things tended to occur with increasing frequency as the years passed, Ray Mindlin would be inconspicuously present among the well-wishers. Perhaps the loyalty and affection made up to some extent for not having any children of his own. He married twice, each time losing a beloved mate by untimely death due to illness.

He served with devotion the profession which he made his life's work, through his research, his teaching, his advisory capacity to numerous government agencies, and his activities in various scientific and technical societies. Among the latter, mention is warranted of the following positions he held at various times: In the American Society of Mechanical Engineers (ASME), chairman, Applied Mechanics Division; member, Publications Committee, Engineering Societies Monographs Committee, Advisory Board of *Applied Mechanics Reviews.* In the American Society of Civil Engineers (ASCE), chairman, Committee on Applied Mechanics of the Structural Engineering Division (precursor of the Engineering Mechanics Division). In the Society for Experimental Stress Analysis (SESA), co-founder and president; member, executive committee. In the American Institute of Physics, associate editor, *Journal of Mathematical Physics.* Also, he was member of the U.S. National Committee for Theoretical and Applied Mechanics; the General Assembly of the International Union of Theoretical and Applied Mechanics; the American Physical Society.

To be sure, his remarkable intellectual fecundity and gentle personality did not go unappreciated by his peers, and recognition came in the form of copious honors and awards. He was elected Fellow of the American Academy of Arts and Sciences (1958), of the ASME (1962), and of the Acoustical Society of America (1963); member of the National Academy of Engineering (1966) and of the National Academy of Sciences (1973); and Honorary Member of the ASME (1969).

He received the Research Prize (1958) and the von Karman Medal (1961) of the ASCE; the Timoshenko Medal (1964) and the ASME Medal (1976) from the ASME; the Trent-Crede Award of the Acoustical Society of America (1971); the Frocht Award of the SESA (1974); the Great Teacher Award (1960) and the Egleston Medal (1971) from Columbia University; and an Honorary D.Sc. degree from Northwestern University (1975); also, a Naval Ordnance Development Award (1945) and the C. B. Sawyer Award of the Army Electronics Command (1967). Remarkably, prestigious as these are, all but one were sandwiched between his most noteworthy honors, both bestowed by the U.S. Government: the Presidential Medal for Merit (1946), as noted earlier the highest civilian decoration of the Second World War, and his ultimate accolade, the National Medal of Science, which he received in 1979.

Mindlin died on November 22, 1987, at age 81, in consequence of a debilitating illness that lingered for about a year. And yet, to the last, his mind remained acute and active, almost as if divorced from his frail body. His passing was mourned by all who knew him, but his name will live through his work.

Chaucer's lines, written about another scholar, of days long gone, might just as aptly have been indited for Raymond Mindlin:

Of study took he utmost care and heed
Not one word spoke he more than was his need
And that was said in fullest reverence
And short and quick and full of high good sense
Pregnant of moral virtue was his speech
And gladly would he learn and gladly teach.

H. Deresiewicz